

1. INTRODUCTION

Quantum-mechanical theory of magnetic monopoles began with the work of Dirac [1]. For a deeper understanding of monopole dynamics Dirac's theory had to be second-quantized. The quantum field theory of electric and magnetic charge (hereafter called QEMD, quantum electro-magnetodynamics) was formulated by Schwinger [2] in a Hamiltonian operator framework. The corresponding Lagrangian formalism was developed by Zwanziger [3] (a two-potential formulation) and Blagojević and Šenjanović [4] (a one-potential formulation). All the three formulations are shown to be equivalent [4],[5].

Despite continuous efforts, the existence of magnetic monopoles has not been confirmed to date [6]. There are, however, theoretical developments which will undoubtedly influence our understanding of experiments.

The 't Hooft-Polyakov extended monopole [7] was obtained as a solution of the $SO(3)$ gauge-invariant unification model. A connection between the extended monopole and Dirac's monopole has been exhibited in Refs. [8]. Bardakci and Samuel [9] have attacked the problem of the effective theory for extended monopoles. They have shown that, in the limit when the size of the extended monopole is shrunk to zero, the effective field theory governing the interaction between extended monopoles is QEMD! The approximation is correct for energies much below the inverse size of the extended monopole. Thus, the study of Dirac's monopole helps us gain a better understanding of extended monopoles, too.

QEMD is a field theory which is not explicitly Poincaré invariant, as it features a preferred direction n_μ , and it possesses two coupling constants interrelated via the quantization condition

$$\ell q / 2\pi = N , \quad (1.1)$$

where N is an integer. This situation makes the study of the consistency of the theory and its relation with experiment rather difficult.

In order for the theory to be consistent it is necessary to confirm Poincaré invariance (n -independence) and solve the problem of ultraviolet and infrared divergences. The first problem was treated by Brandt, Neri and Zwanziger [10]. Important progress was also made in ultraviolet regularization [11], and the infrared problem was formally solved using the one-potential formulation [12],[5].

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SOFT PHOTON RADIATION EFFECTS IN MONPOLE PROCESSES *

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ABSTRACT

The infrared problem of the quantum field theory of monopoles and charges is reviewed and discussed in explicit terms. It is shown that the related radiation effects yield a superstrong damping of the cross-section in the relativistic kinematic region, which leads to a possible mechanism for partial monopole confinement.

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In this article we will give a review of the one-potential formulation of QEMD, the solution of the infrared problem and the related superstrong radiation damping effects that go beyond naive perturbation theory, and discuss a possible dynamical mechanism of (partial) monopole confinement.

$$-\partial^\mu (G_{\mu\nu} - G_{\nu\mu}) = j_\nu^g .$$

It can be satisfied by the ansatz

$$G_{\mu\nu} = h_\mu j_\nu^g , \quad (2.5a)$$

2. THE ONE-POTENTIAL FORMULATION OF QEMD

The investigation of the basic properties of the theory is facilitated by the existence of various formulations. The formulation we are going to describe represents a natural generalization of Dirac's quantum-mechanical theory to quantum field theory. It is based on a Lagrangian and features only one four-potential [4],[5].

A. Classical Lagrangian formalism

We are after a Lagrangian that will yield the generalized Maxwell equations:

$$\partial_\mu F^{\mu\nu} = j_\nu^e , \quad (2.1)$$

$$\partial_\mu *F^{\mu\nu} = j_\nu^g . \quad (2.2)$$

Here, $*F$ means the tensor dual to F , j_e and j_g are the conserved electric and magnetic currents, respectively,

$$j_\nu^e = e \bar{\psi} \gamma^\nu \psi , \quad j_\nu^g = g \bar{\chi} \gamma^\nu \chi , \quad (2.3)$$

and ψ and χ are spin 1/2 fields describing a pure charge and a pure monopole (the generalization to dyons is direct), respectively.

Since the usual way of introducing the electromagnetic potential by setting $F = \partial \wedge A$ leads to $\partial *F = 0$, in contradiction to Eq.(2.2), we attempt

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \epsilon_{\mu\nu\lambda\sigma} G^{\lambda\sigma} . \quad (2.4)$$

Eq. (2.2) produces a condition on the tensor G

$$[\chi \cdot (i\partial - eA) - m_1] \chi = 0 , \quad (2.7c)$$

provided the magnetic current j_g is conserved and $\partial \cdot h = -\delta$. We are free to choose h as

$$h^\mu(x) = -\eta^\mu(n \cdot \partial)^{-1}(x) ,$$

$$(\eta \cdot \partial)^{-1}(x) = [-\alpha \theta(n \cdot x) + (1-\alpha)\theta(-n \cdot x)] \delta_n(x) , \quad (2.5b)$$

where n^μ is a constant vector, $\theta(u)$ is the one-sided step function, α is an arbitrary real number, and $\delta_n(x)$ is a three-dimensional δ -function with support on the hypersurface through the origin, whose normal is n^μ .

The Lagrangian for the theory is taken to be

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F^2 + \bar{\psi} [\chi \cdot (i\partial - eA) - m_1] \psi \\ & + \bar{\chi} (\chi \cdot i\partial - m_2) \chi . \end{aligned} \quad (2.6)$$

We observe an asymmetry here in the description of electric and magnetic variables, but this is only apparent. The Lagrangian (2.6) leads to the following equations of motion:

$$\partial_\mu F^{\mu\nu} = j_\nu^e , \quad (2.7a)$$

$$[\chi \cdot (i\partial - eA) - m_1] \psi = 0 , \quad (2.7b)$$

where the potential B depends on other variables $B^\nu = h_\mu^T F^{\mu\nu}$. The second Maxwell equation is identically satisfied with the choice (2.5) for G . Eqs. (2.7b,c) lead to the conservation of electric and magnetic currents, which is the consistency condition for Maxwell's equations.

We shall choose n^μ to be a spacelike vector so that we can have a choice $n^0 = 0$ for which the theory is local in time. The above formulation has turned out to be a field-theoretic counterpart of Dirac's quantum-mechanical theory, with a frozen string lying in the direction of the vector n^μ [13].

B. Feynman rules

The classical theory (2.6) can be quantized by standard techniques [4], [5]. The perturbation expansion in a theory of this type possesses features that render it formal: (a) it contains two coupling constants which turn out not to be independent – the demand of Lorentz invariance relates one to the other, Eq. (1.1); (b) one of the coupling constants (the magnetic one) is very large; (c) to any finite order in perturbation expansion the theory is not Lorentz invariant [10]. Nevertheless, the (formal) Feynman rules will be useful in giving us insight into the structure of the theory.

It is useful to first transcribe the Lagrangian (2.6) into the form

$$\mathcal{L} = -\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - j^\mu A_\mu - \frac{1}{2} j_\mu^\lambda (n^2 \eta_{\lambda\sigma} - \eta_{\lambda} \eta_{\sigma}) (n \cdot \partial)^2 j_\mu^\sigma - \epsilon^{\mu\nu\lambda\sigma} (n \cdot \partial)^4 \partial_\mu A_\nu n_\lambda j_\mu^\sigma + \bar{\psi} (\imath \hat{\partial} - m_1) \psi + \bar{\chi} (\imath \hat{\partial} - m_2) \chi .$$

From this expression and a simple analysis we obtain the following Feynman rules, local in momentum space (Fig. 1):

$$D_F^{\mu\nu} = -i \eta^{\mu\nu} / k^2 , \quad (2.8a)$$

the photon-charge vertex

$$V_e^\nu = -i e j^\nu , \quad (2.8b)$$

the photon-pole vertex

$$\Gamma_\gamma = -i g \epsilon_{\mu\nu\lambda} k^\mu n^\lambda j^\sigma / n \cdot k \equiv -i g A_\nu j^\sigma . \quad (2.8c)$$

The four-pole vertex Λ can be simulated by two naive vertices of the type $V_g^\mu = -ig \gamma^\mu$ connected by a spurion propagator D_s

$$\begin{aligned} \Lambda &= V_g^\mu [-i (n^2 \eta_{\mu\nu} - \eta_\mu \eta_\nu) / (n \cdot k)^2] V_g^\nu \\ &\equiv V_g^\mu D_{\mu\nu}^S V_g^\nu . \end{aligned} \quad (2.8d)$$

In all considerations concerning the contribution of diagrams containing photon exchange between two monopole lines, calculations can be simplified substantially. The Feynman diagram structure clearly points to the dual symmetry of the theory [14]. Two graphs involving photon and spurion exchange add up to a graph with naive vertices V_g connected by the effective propagator D_E (Fig. 2)

$$\begin{aligned} \Gamma D_F \Gamma + V_g D_S V_g &= V_g D_E V_g , \\ D_E^{\mu\nu} &= -\frac{i}{k^2} \left[\eta^{\mu\nu} - \frac{n^\mu k^\nu + n^\nu k^\mu}{n \cdot k} - \frac{n^2}{(n \cdot k)^2} k^\mu k^\nu \right] , \end{aligned} \quad (2.9)$$

with D_E satisfying $n \cdot D_E = 0$. The n -dependent terms in the effective propagator can be gauged away. Indeed, Zwanziger's two-potential formulation with the gauge fixing condition $n \cdot B = 0$ leads precisely to this propagator in the magnetic sector of the theory. A different choice of the gauge-fixing procedure, say, adding a gauge breaking term $-(\partial \cdot B)^2/2$, will lead to

$$D'_E^{\mu\nu} = -i \eta^{\mu\nu} / k^2 , \quad (2.10)$$

in conformity with the dual symmetry of the theory.

3. INFRARED REGULARIZATION

If the Feynman rules in QED are used to calculate higher-order radiative corrections for a process involving charged particles and a definite number of external photons, there arise divergences stemming from soft (i.e. low-frequency) virtual photons. The problem can be solved by considering a realistic experiment in which real photons may be emitted, which are too soft to be detected; it is the energy resolution Δ of the apparatus which determines

whether such photon emission is recorded or not. The experimentally observed cross-section always includes the bremsstrahlung, i.e. the emission of soft photons, of energy less than Δ . The contribution of the real soft photon emission to the cross-section is also infrared divergent, but this divergence is exactly cancelled by the infrared divergence stemming from virtual soft photons. Hence, the experimental cross-section remains finite and depends on the energy resolution Δ .

The origin of the infrared problem lies in treating soft bremsstrahlung and higher-order corrections as separate processes in perturbation theory. It is directly related to the masslessness of the photon, and so it also appears in QEMD. The solution of the problem in QEMD is analogous to the one in QED [12]. The analogy is most explicit in the one-potential formulation.

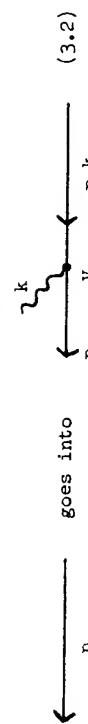
A. Charge potential scattering

To illustrate the nature of the infrared problem, let us first consider an arbitrary diagram D describing the potential scattering of an electron in QED, Fig. 3 (the shaded area involves external fields as well as internal and external photon lines, and internal electron lines). The matrix element corresponding to D will be of the form

$$M_0 = \bar{U}(p') \Gamma(p, k_i) U(p) , \quad (3.1)$$

where Γ is associated with the shaded area and k_i is the momentum transferred at the i -th vertex.

a) Virtual photon corrections. We wish to calculate radiative corrections to the basic process M_0 . Let \tilde{D}_V denote the set of diagrams obtained from D by inserting one additional virtual photon line into D in all possible ways. The insertion of one end of the additional photon line into a given (internal or external) electron line is effected by the replacement



One can show that the leading infrared behaviour in the momentum k stems from diagrams in which both ends of the additional photon line are attached to external electron lines [15]. Indeed, if p is the momentum of the external line ($p^2 = m^2$) then the additional electron propagator $G_{(p-k)}$ will be close

to its pole for small k , as $(p-k)^2 \approx 0$. [Diagrams in which at least one end of the additional photon line terminates on an internal electron line may have overlapping infrared divergences in k and k_i , which arise when $k \rightarrow 0$ and $k_i \rightarrow 0$ simultaneously, but they are seen to cancel.] Therefore, the leading infrared contribution of \tilde{D}_V is given by diagrams shown in Fig. 4. The corresponding amplitude can be written in the form

$$M_V = M_0 \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} R^\mu \tilde{D}_F^{\mu\nu} R_\nu \equiv M_0 \alpha_e B . \quad (3.3)$$

Here, R^μ is the infrared factor of virtual photons associated with the electron line

$$R_\mu^\mu = -ie T_\mu(p', p) \equiv -ie \left(\frac{2p'_\mu - k_\mu}{2p \cdot k - k^2} - \frac{2p_\mu - k_\mu}{2pk - k^2} \right) , \quad (3.4)$$

$\alpha_e = e^2/4\pi$, and \tilde{D}_F is the photon propagator,

$$\tilde{D}_F^{\mu\nu} = -i\eta^{\mu\nu}/(k^2 - \lambda^2) \equiv -i\eta^{\mu\nu} \tilde{\Delta}_F .$$

We have assigned a fictitious small mass $\lambda (\neq 0)$ to the photon in order to regularize all divergences; at the end of the calculation we shall take the limit $\lambda \rightarrow 0$ to regain QED. The multiplicative infrared factor B in the amplitude (3.3) is given by

$$B = \frac{i}{8\pi^3} \int \frac{d^4 k}{k^2 - \lambda^2} I_\mu(p', p) I^\mu(p', p) \equiv B(p', p) . \quad (3.5)$$

Summing the basic amplitude M_0 with the infrared radiative corrections M_V leads to

$$M' = M_0 (1 + \alpha_e B) .$$

The corrections of order α_e to the cross-section are obtained from the cross term between M_0 and M_V :

$$d\sigma' = d\sigma_0 [1 + 2\alpha_e \text{Re } B + O(\alpha_e^2)] . \quad (3.6a)$$

We see that the contribution of an additional virtual soft photon is factorizable. This property can be easily generalized to include an arbitrary number of virtual soft photons: to each such photon there corresponds a multiplicative factor of the same form, and one should also include the factors $1/n!$ to take care of the statistics. The summation over n gives [15]

$$d\sigma' = d\sigma_0 \exp(2\alpha_e \operatorname{Re} B) . \quad (3.6a)$$

This is true for each diagram of the type D, Fig. 3, and therefore for the complete process of the electron potential scattering.

It is interesting to note that $\operatorname{Re} B \rightarrow -\infty$ as $\lambda \rightarrow 0$, hence the probability of a given process vanishes. Only when we include the emission of an infinite number of real soft photons the cross-section will be nonvanishing. This is in agreement with the result of classical electrodynamics that an accelerated charge must radiate.

b) Real photon corrections. To calculate the contribution of soft photon emission, let us consider the set of diagrams \tilde{D}_R obtained from D by inserting an additional real photon line into D in all possible ways. The overlapping divergences cancel in the same manner as for virtual photons so that only photons terminating on external electron lines give the infrared contribution, Fig. 5. The corresponding amplitude takes the form

$$M_R = M_0 \bar{R}_\mu^e \epsilon^\mu , \quad (3.7)$$

where \bar{R}^e is the infrared factor of real photons associated with the electron line

$$\bar{R}_\mu^e = -ie \bar{T}_\mu(p', p) \equiv -ie \left(\frac{p'_\mu}{p \cdot k} - \frac{p_\mu}{p \cdot k} \right) . \quad (3.8)$$

To obtain the cross-section we square the amplitude, sum over photon polarizations ϵ_μ and integrate over its momentum up to the experimental threshold Δ ,

$$d\sigma_R = d\sigma_0 \int_0^\Delta \frac{d^3k}{(2\pi)^3 2\omega_k} \bar{R}_\mu^e \bar{R}_\mu^e = d\sigma_0 2\alpha_e \bar{B} . \quad (3.9)$$

Here $\omega_\lambda = (\vec{k}^2 + \lambda^2)^{1/2}$, and

$$\bar{B} = -\frac{i}{8\pi^2} \int_0^\Delta \frac{d^3k}{\omega_\lambda} \bar{T}_\mu(p', p) \equiv \bar{B}(p', p) . \quad (3.10)$$

Adding this part to the earlier result (3.6a) we obtain the complete cross-section including soft photon infrared corrections of order α_e relative to the basic process,

$$d\sigma = d\sigma_0 [1 + 2\alpha_e (\operatorname{Re} B + \bar{B}) + O(\alpha_e^2)] . \quad (3.11a)$$

The generalization to an arbitrary number of (real and virtual) photons and the corresponding summation lead to the general result

$$d\sigma = d\sigma_0 \exp [2\alpha_e (\operatorname{Re} B + \bar{B})] , \quad (3.11b)$$

where $d\sigma_0$ is independent of the soft photon limit, and $\operatorname{Re} B$ and \bar{B} represent infrared contributions from the lowest order radiative corrections.

The infrared contribution to B can be calculated by using the formula

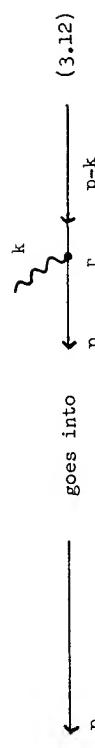
$$\frac{1}{k^2 - \lambda^2 + i\epsilon} = PV \frac{1}{k^2 - \lambda^2} - i\pi \delta(k^2 - \lambda^2) ,$$

resulting in the complete cancellation of infrared (λ -dependent) divergences in $\operatorname{Re} B + \bar{B}$.

B. Monopole potential scattering

Let us now turn back to QM&D and consider the monopole potential scattering as the simplest case, which will enable us to gain insight into the infrared problem of processes including monopoles. An arbitrary diagram corresponding to this process can be represented as in Fig. 3, but now the external fermion lines are monopole lines.

a) Virtual photon corrections. The insertion of one end of the additional photon line into a given monopole line is realized by the substitution



When the additional photon line is virtual, the infrared factor associated with the monopole line is given by

$$R^g_\mu = -iq A_{\mu\nu}(\mathbf{k}) I^\nu(p', p) . \quad (3.13)$$

This expression differs from the QED one (R_μ^e) by the presence of the factor $A_{\mu\nu}$, originating from the different photon-pole vertex. Diagrams in which both ends of the additional photon line are attached to external monopole lines (Fig. 4) give an amplitude of the form

$$M_0 \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} R^g_\mu D_F^{\mu\nu} R^g_\nu .$$

We now observe that, in QED, in addition to photon exchange diagrams (Fig. 4) there are always the corresponding spurion exchange diagrams (Fig. 6). This greatly simplifies further considerations! Combining photon and spurion exchange and using Eqs. (2.9) and (2.10), we find

$$\begin{aligned} R_\mu^e D_F^{\mu\nu} R_\nu^e &+ \text{spurion exchange} \\ &= ig^2 I_\mu^{\nu\mu} \Delta_F \end{aligned} \quad (3.14)$$

We now see that virtual photon and spurion radiative corrections lead to the same amplitude

$$M'_\nu = M_0 \alpha_g B(p', p) , \quad (3.15)$$

which is the same as in QED up to the replacement $e^2 \rightarrow g^2$, in agreement with the dual symmetry between electric and magnetic sectors of the theory ($\alpha_g = g^2/4\pi$).

Diagrams in Figs. 4 and 6 are the only ones that produce the radiative infrared corrections to the basic monopole scattering process.

b) Real photon corrections. The infrared factor representing the amplitude for emission of real soft photons from monopole line is

$$\bar{R}^g_\mu = -iq A_{\mu\nu}(\mathbf{k}) I^\nu(p', p) . \quad (3.16)$$

Potential string singularities at $n \cdot k = 0$ are present in $A_{\mu\nu}$, but we can get rid of those easily. Indeed if we square \bar{R}_g^2 to produce the emission cross-section, we obtain

$$2(R e B + \bar{B}) = K \ln 2\Delta/m + K_1 + K_2 . \quad (3.21)$$

C. Monopole-charge scattering

For heavy monopoles the potential scattering picture is quite inadequate, so we shall now consider the infrared problem in monopole-charge scattering. When compared to monopole potential scattering, here we have additional soft photon contributions from the charge line, and virtual photon contributions stemming from photon (and spurion!) exchange between monopole and charge lines. Using methods analogous to the one in potential scattering, we obtain the cross-section of the form (3.19), but with B and \bar{B} replaced by

$$q^2 B \rightarrow q^2 \bar{B}(p_4, p_2) + e^2 B(p_3, p_1) + eq F ,$$

$$q^2 \bar{B} \rightarrow q^2 \bar{B}(p_4, p_2) + e^2 \bar{B}(p_3, p_1) + eq \bar{F} , \quad (3.22)$$

where F and \bar{F} describe mixed $e\text{-}g$ contributions. The terms proportional to g^2 and e^2 in B and \bar{B} combine to yield an infrared finite result. The terms proportional to eg , which appear n-dependent due to the presence of $A_{\mu\nu}(k)$ term in monopole-photon vertex, are in fact equal to zero [16], [5]. This result is very important as it ensures n-independence of the infrared contribution to the process.

D. Monopole pair creation

Pair creation complements scattering process in giving us a deeper insight into monopole dynamics. The case of pair creation in a space- and time-dependent external field, which is simple for calculation, should provide many features of a more general case of pair creation from magnetically neutral initial state. The treatment of the infrared problem is analogous to the case of potential scattering, as this process lies in just another kinematic region of the former, characterized by the substitution $p' \rightarrow -p'$ (see Fig. 3).

The differential cross-section with energy loss Δ is given by expression (3.19), but with $p' \rightarrow -p'$.

In solving the infrared problem in QEMD, we have followed the method of Yenni, Frautschi and Sura [15], in which the infrared finiteness of the physical cross-section is proved in the realm of perturbation theory. An alternative approach via coherent states in QED was provided by Chung [17]. QEMD was also shown to be infrared finite by use of coherent states [18].

4. RADIATION DAMPING EFFECTS

We have seen that infrared divergences in monopole processes cancel, i.e. $ReB + \bar{B}$ remains finite as $\lambda \rightarrow 0$. The effects of this finite exponentiated piece (stemming from soft photons) in the cross-section are very small in QED due to the smallness of the fine structure constant. In QEMD such effects can be enormous since the relevant coupling constant is large.

In order to see the nature of this effect more closely, let us first look at monopole potential scattering. In the relativistic kinematic region $|(p' - p)^2| > m^2$, and $\Delta \ll E$, one finds [15]

$$Re B + \bar{B} \approx -\frac{1}{4} K \ln EE'/\Delta^2 + \frac{1}{8} K , \quad (4.1)$$

where $K = (2/\pi)(\ln 2p'p/m^2 - 1)$, E and E' are monopole energies before and after scattering. This result means a superstrong damping of the cross-section with a factor smaller than $\exp(-1.37)$! In the nonrelativistic region nonfactorizable radiation effects, which at present we cannot calculate, must not be neglected. Therefore, in this region the calculations leading to the simple form of the cross-section as in Eq.(3.19) are not to be trusted.

In monopole-charge scattering the damping has two parts, one multiplying e^2 , the other g^2 [see Eq.(3.22)]. Both parts are of the same type as in the case of monopole potential scattering.

In considering the scattering process we used the laboratory reference system (threshold energy Δ is defined in a system where the detector is at rest, i.e. in the laboratory system); in pair-creation in an external field it is natural to use the center-of-mass system. Defining a suitable variable $u = (p'p'/2m)^{1/2}$, one finds that in the relativistic kinematic region, $u \gg 1$, the asymptotic behaviour of $ReB + \bar{B}$ is given by

$$Re B + \bar{B} \approx -\frac{1}{\pi} (\ln 4u^2 - 1) \ln \frac{m}{2\Delta} - \frac{1}{\pi} (\ln 4u^2) \ln u , \quad (4.2)$$

and we again observe an enormous damping effect. The behaviour of $ReB + \bar{B}$ is presented in Fig. 7 for two values of $2\Delta/m$ (10^{-2} and 10^{-18}). In both cases $ReB + \bar{B}$ is negative in the whole region $u > 1$, and causes significant damping.

For small values of u (below $u=1$), the expression for $\text{Re}\vec{B} + \vec{E}$ cannot be trusted for the following reasons: a) we have not taken into account nonfactorizable radiation effects which are small in the region $u \gg 1$, but this is not true for small u ; b) the effect of the interchange of bound states begins to interfere below $u = 2m$ [19].

In the complete expression for the cross-section, Eq.(3.19), the contribution of soft photon radiation is factorized. We have found that the part $\text{Re}\vec{B} + \vec{E}$ causes significant damping in the cross-section in the relativistic kinematic region, where the approximation of factorizable radiation can be trusted. The presence of an additional factor in the complete cross-section, which is an unknown function of e and g (g is a large coupling constant), demands some caution. We cannot rule out the possibility that this factor may counterexponentiate and cancel, or completely overshadow the damping in the opposite direction. Yet, since there is no physical reason for this to occur, the effect is likely to be genuine.

These results are also supported by the result of Drukier and Nussinov [20], who demonstrated such an exponential suppression for pair production of extended monopoles. In view of the relation between extended monopoles and QEMD, the two results strengthen each other.

We have seen that the leading infrared contribution is n-independent. However, there are also infrared convergent photon contributions to the cross-section, which are n-dependent (due to the factor $A_{\mu\nu}$ in photon-pole vertex) and may have singular behaviour for $n \cdot k = 0$. We shall call such possible infinities string infinities. The proof that gauge-invariant Green's functions in QEMD are n-independent [10] is carried out by using the relation $\exp(ieg) = 1$, which follows from the quantization condition (1.1). This relation can never hold in any finite order perturbation expansion. It is clear, therefore, that n-dependent parts of potential string infinities may cancel only after summation of all diagrams. By considering extended monopoles and their connection with QEMD [9] one can see that such a singularity must be artificial, as there is a gauge in which no string appears in the classical monopole solution.

5. PARTIAL MONPOLE CONFINEMENT

It has been conjectured that partial (or total) confinement of magnetic monopoles should occur on the following grounds: as the bound monopole and antimonopole separate in order to be liberated, they radiate profusely losing a large amount of energy on radiation, whereupon, as they are slowed down rapidly, they retract [21]. On the other hand, we have calculated such radiation effects, stemming from soft photons, and found a huge damping of the pair production cross-section in the relativistic kinematic region. This naturally leads to the question as to whether such effects exist in liberation from magnetically neutral bound states [16] (Fig. 8).

To answer that question, let us consider bound states of a spin-1/2 and a spin-0 dyon. The dynamics is given by

$$\mathcal{L} = \mathcal{L}_Y + \mathcal{L}_\phi + \mathcal{L}_\psi , \quad (5.1)$$

where \mathcal{L}_Y is the Lagrangian of the free electromagnetic field, and \mathcal{L}_ϕ and \mathcal{L}_ψ are the Lagrangians of dyons of spin 0 and spin 1/2, respectively, with minimal coupling to the electromagnetic potentials. Here we assume the existence of stable bound states, magnetically neutral, of spin 1/2 ($e_1 + e_2 = e$, $g_1 + g_2 = 0$, $s_1 + s_2 = 1/2 + 0$). We take the case when dynamical spin is absent [5]. The formalism we use in the treatment of bound states is the Bethe-Salpeter equation [22], which, in the case considered, assumes the form

$$(\mathcal{D}_1 + V_1) (D_2 + V_2) = \bar{G} \chi . \quad (5.2)$$

Here, \mathcal{D}_1 and D_2 are the Dirac and the Klein-Gordon operators, respectively, χ is the Bethe-Salpeter amplitude of the bound state, \bar{G} is the irreducible kernel of the system, and the electromagnetic interactions V_1 and V_2 are given as

$$\begin{aligned} V_1 &= -e_1 \chi_\mu A_1^\mu , \\ V_2 &= -ie_2 (\partial_2^\mu A_{2\mu} + A_{2\mu} \partial_2^\mu) - (e_2)^2 A_{2\mu} A_2^\mu . \end{aligned}$$

We have temporarily deleted magnetic vertices, which can be treated in the same manner.

With this as the starting point in the process of generating effective vertices, we encountered renormalizable and nonrenormalizable ones. Except for one kind of nonrenormalizable vertex, all the others were infrared convergent, while the one contributing to the infrared regime was seen to be suppressed in the tight-binding limit. The discussion of effective vertices shows that composite states can be replaced by effective point-like electrically charged states. Thus, compositeness does not affect the infrared behaviour, and we obtain the same sort of radiation damping as in the previous section. This conclusion confirms the conjecture of the partial monopole confinement in the relativistic kinematic region.

6. CONCLUSIONS

In this paper we have studied the infrared behaviour of QEMD using the one-potential formulation. It is shown that the leading infrared contribution is factorizable and can be exponentiated with a cancellation of infrared divergent contributions of real and virtual soft photons.

The remaining infrared finite parts of soft photon contributions yield a significant damping of the cross-section in the relativistic kinematic region, where, moreover, we can trust the approximations made. It is also shown that the same sort of radiation damping is obtained in monopole pair production from magnetically neutral bound states, which supports the conjecture of partial monopole confinement. The radiation damping factor is shown to be n-independent - a property that must be satisfied by any physical prediction of the theory.

The physical implications of the phenomenon of superstrong radiation damping are the following: a) As in scattering the damping is enormous in the relativistic kinematic region, experimental searches should be geared toward detection of slow monopoles, and since these get stopped in the atmosphere more easily, this would seem to suggest the need for satellite observation. b) Our results do not clarify the case of monopoles in the early universe, as the kinematic region in this case is $u \ll 1$ ($M \approx 10^{17}$ GeV, $T_c \approx 10^{15}$ GeV). However, our analysis shows that the Born approximation may be misleading, and this should serve as a work of caution [23]. c) Finally, radiation damping has a bearing on the problem of monopole confinement.

The progress made in treating n-independence, infrared problem and renormalization [24] is an important step in establishing QEMD as a well rounded (effective) quantum field theory of interacting monopoles and charges.

ACKNOWLEDGMENTS

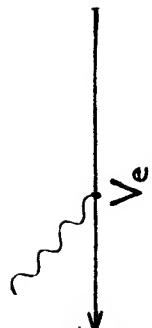
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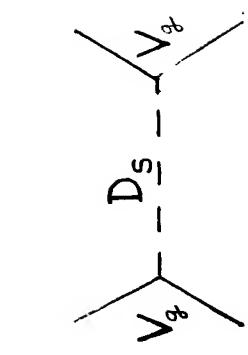
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- [24] It has been shown in Ref. [11] that the existence of the Wess-Zumino type interaction in the particle-path representation of QEMD yields $Z\epsilon g = 1$. The result is obtained in an n-independent procedure.

FIGURE CAPTIONS

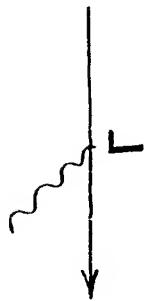
(a)



(b)



(c)



(d)

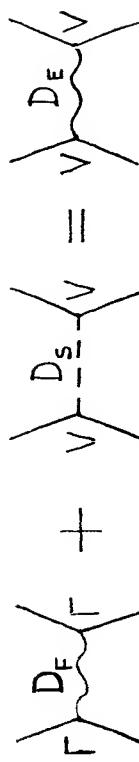


FIG. 1.

Fig. 1 The Feynman rules of the one-potential formulation:

- a) the photon propagator
- b) the photon-charge vertex
- c) the photon-pole vertex
- d) the spurion propagator simulating the four-pole vertex.

Fig. 2 Exchange of a photon together with one of a spurion results in the effective propagator.

Fig. 3 The basic diagram describing the potential scattering.

Fig. 4 Diagrams with one additional virtual photon which contributes to infrared divergences.

Fig. 5 Diagrams with one additional real photon which contribute to infrared divergences.

Fig. 6 Diagrams involving spurion exchange also contribute to infrared divergences.

Fig. 7 Behaviour of $R\epsilon B + \bar{B}$ for (a) $2\Delta/m = 10^{-2}$ and (b) $2\Delta/m = 10^{-18}$.

Fig. 8 Monopole pair production from magnetically neutral bound states.

FIG. 2.

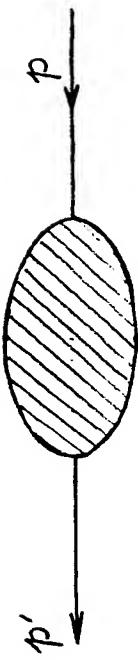


FIG. 3.

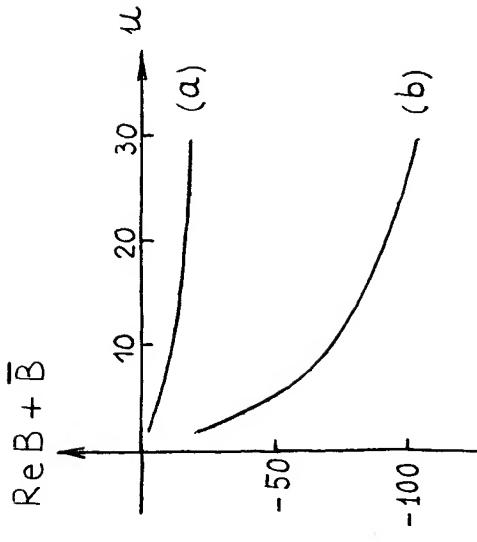


FIG. 4.

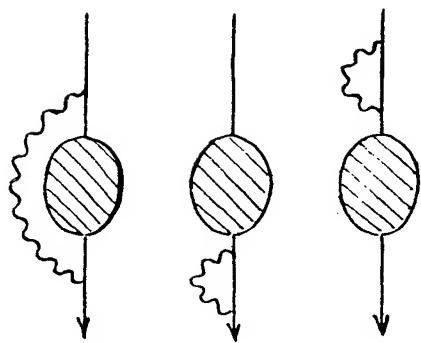


FIG. 5.

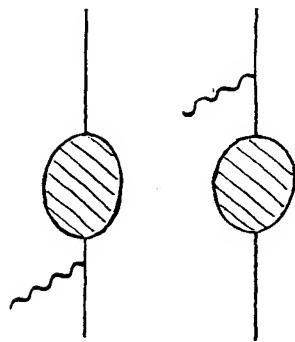


FIG. 6.

$$\begin{aligned} D &\geq \frac{\pi^2 N_0 l^2}{4} \quad \text{for } k \neq 0 \\ \text{or } k &\neq 0 \quad f = \left(N_0, \frac{\pi}{l} \right) \in \mathbb{Z}^2 \\ \text{if } k &\neq 0 \quad \alpha = 0 \end{aligned}$$

FIG. 7.

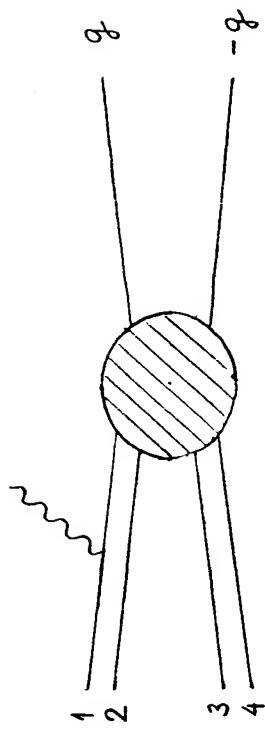


FIG. 8.